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Using Genetic Algorithms in Designing Substation Lightning Shielding

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***Abstract*—This paper introduces application of genetic algorithm (GA) in designing air-termination system of the high-voltage open-air substation external lightning protection system (LPS). The air-termination system, as a part of the external LPS of high-voltage substation, is known to be cumbersome to design using traditional techniques, where optimization is often relegated to the trial-and-error approach. In addition, traditional methods are often associated with geometric constraints and various implementation difficulties. This paper proposes a novel approach which utilizes a combination of statistical LPS efficiency and GA in designing techno-economically optimal external LPS of open-air substations. Proposed approach offers a LPS designer unique and valuable assistance in optimally arranging elements of air-termination system for obtaining maximum lightning shielding effects with minimum total investments.**

Keywords—Genetic algorithm; Lightning protection; LPS; Substation shielding; EGM

I. INTRODUCTION

This paper proposes application of genetic algorithm (GA) in designing air-termination system of the high-voltage open-air substation lightning protection system (LPS). As far as the authors are informed, this is the first proposal for using GA in designing substations lightning shielding. The air-termination system is known to be cumbersome to design using traditional techniques, where optimization is often relegated to the trial-and-error approach; see IEEE Std. 998-2012 and IEC 62305-2 for more information. In addition, traditional methods are often associated with geometric constraints and implementation difficulties [Hileman, 1999].

This paper proposes a novel approach which utilizes a combination of statistical LPS efficiency and GA in designing techno-economically optimal substation lightning shielding. This optimization problem is particularly well suited to the GA solution approach, due to the stochastic nature of lightning, expert knowledge involved in constructing the fitness function, and the fact that the optimal solution is not, strictly speaking, a unique value (it can be obtained using different combinations of the air-termination system elements).

In creating LPS shielding zones, station designer needs to propose some initial arrangement of shielding conductors and apply one of the methods from IEEE Std. 998-2012. If the derived protection zones do not encompass equipment, for a preselect lightning-current level and/or shielding failure rate, designer needs to make changes to the disposition of air-terminations (change position and/or height of existing and/or add additional horizontal/vertical conductors) and conduct another analysis. This process of positioning and re-positioning of air-terminations is essentially a trial-and-error process, with no certain guarantees that the re-positioning of the parts of the air-termination system, between successive iteration steps, will actually improve the overall LPS design [Chowdhuri, 1996]. In other words, there is no comprehensive optimization involved in the traditional approach to the LPS design. Hence, the main purpose of this paper is to address this deficiency.

The aim of this paper is to recast the problem of the LPS design as an optimization problem, and to further approach it from the statistical perspective in order to fully account for the stochastic nature of lightning. For that purpose, large number of lightning strikes are simulated by means of the Monte-Carlo method [Sargent, 1972; Vujevic et al., 2008a; Vujevic et al., 2008b; Srivastava and Mishra, 2015]. Each lightning strike starts its descend from a plane above the substation (i.e. fictitious cloud base) and follows a stochastic path towards the earth surface. Minimal distance from the lightning stroke stepped leader head to elements of LPS, equipment, and earth surface is computed after each jump. If this distance is smaller than the “striking distance,” in accordance with the EGM, lightning will strike its nearest point belonging either to the LPS, station equipment, or the earth surface. Stochastic efficiency of the LPS is obtained from a quotient between the number of lightning strikes ending up on the LPS itself and the total number of lightning strikes (where strikes hitting the earth surface are discarded).

In addition, on top of the Monte-Carlo method, genetic algorithm is employed to guide the LPS design toward the optimal disposition of air-terminations, i.e. that which has the highest stochastic efficiency with minimal design investments. Three different GA operators are used in order to produce offspring [Goldberg, 1989]: crossover, mutation and reproduction. Selection of individuals from the parental population is carried out using a tournament selection principle. Elitism is also applied. A particular emphasis will be given to the construction of the fitness function, using expert knowledge on lightning protection and careful adjustments to the nature of the optimization problem at hand, as well as to the particular chromosome's representation.

Proposed approach offers a LPS designer unique and valuable assistance in optimally arranging elements of air-termination system for obtaining maximum lightning shielding effects with minimum total investments. It will be demonstrated on a concrete open-air high-voltage transformer station.

II. STOCHASTIC EFFICIENCY OF LIGHTNING SHIELDING

For the purpose of the substation LPS design, only downward negative lightning current amplitudes are of importance [CIGRE, 2013]. Hence, a large number of pseudo-random lightning current amplitudes are drawn from the appropriate Log-Normal distribution, and then arranged into an arbitrary number of classes [Vujevic at al., 2008a; Vujevic et al., 2008b]. The AIS equipment and external LPS elements are represented by straight segments (or triangles in case of surfaces) within the global Cartesian coordinate system, such that the xy-plane (z=0) coincides with the earth surface. A large number (N) of successive lightning strikes is initiated from the starting surface A_0, representing cloud base and located on some height above the earth surface (centered above the AIS), satisfying the condition $A_0 << A_d$, where A_d is the "collection area" of the AIS [CIGRE, 1997]; see also IEC 62305-2 for more information. It is known that the lightning stroke development, i.e. stepped leader descent, follows a number of quick jumps along a stochastic path. During the Monte-Carlo simulation, each lightning strike starts its descent at some point stochastically chosen on the surface A_0 and progresses downward in a series of stochastic jumps towards the AIS or the earth surface [Vujevic at al., 2008a; Vujevic et al., 2008b].

In order to determine where the stepped leader head is going to end up, one needs to compute the minimal distance between the stepped leader head and a nearest point on the air-termination system, on the AIS equipment, or on the earth's surface. Computing the shortest distance between a point that represents a stepped leader head and a segment in a Cartesian coordinate system involves analytic geometry in 3D space. The interested reader is at this point advised to consult [Vujevic at al., 2008a; Vujevic at al., 2008b] for the in-depth treatmet and more information on computing lightning step-leader jump distance and its termination points on the elements of LPS and substation equipment. It should be mentioned that the EGM of lightning attachment has been employed in the computation of the striking distances and that effects of answering streamers to approaching stepped leader have been neglected [Hileman, 1999; Vujevic at al., 2008b].

The stochastic efficiency of the air-termination system of the external LPS, for the arbitrary i-th class of lightning current amplitudes, can be determined as follows:

$$E_i = \frac{N^i_{LPS}}{N^i_{LPS} + N^i_{AIS}} \tag{1}$$

where N^i_{LPS} and N^i_{AIS} are, respectively, cumulative numbers of lightning strikes to the elements of LPS and AIS equipment. They are obtained by simply counting the number of strikes to each of the elements (from each lightning class). The overall (total) stochastic LPS efficiency can be determined using the following expression:

$$E_{tot} = \frac{\sum_{i=1}^{N_c} P(I^i_{min}) \cdot N^i_{LPS}}{\sum_{i=1}^{N_c} P(I^i_{min}) \cdot \left(N^i_{LPS} + N^i_{AIS}\right)} \tag{2}$$

where $P(I^i_{min})$ stands for the complementary cumulative distribution function of the Log-Normal distribution (of the minimum lightning current amplitude), within each of the lightning classes, and assures that the first few classes, associated with lower lightning current values, contribute more to the overall stochastic LPS efficiency. First few lightning classes (with low amplitudes) account for the majority of shielding failures and these are, hence, given higher weights in calculating the overall stochastic LPS efficiency.

III. GENETIC ALGORITHM DESCRIPTION

In designing the air-termination system of the external LPS of AIS, design engineer is presented, either with existing or planned, AIS equipment disposition, which imposes certain limitations and restrictions (e.g., crane access zones, fire zones, roads) on the possibilities of the LPS realization. Added to that, certain parts of the external LPS will be fixed and not amenable to the design evolution (optimization), such as the grounded portal towers of the incoming transmission lines. Also, there is often a finite number of possible candidate spots for installing air-termination system elements which satisfy primary station design criteria. These will form a variable part of the external LPS design which can be optimized. In addition, if company policy allows, initial LPS design proposal can also include shield wires. Consequently, the variable part of the external LPS will mainly consist of lightning rods (i.e. vertical conductors) installed on grounded parts of the station construction or (sometimes) erected as a free-standing masts. The length of these lightning rods can be optimized in order to provide maximum shielding effects with a minimum total (cumulative) length and without any of the rods being excessively long. Satisfying these conditions is seen here as providing for the techno-economically optimal design.

Each individual (i.e. chromosome), which represents a concrete LPS design solution, in terms of the particular lengths of the lightning rods (i.e. vertical air-termination conductors), is represented with a real vector of N_r elements, where N_r is the number of lightning rods that form a variable part of the external LPS design. Each element of this vector is a random number drawn from the Normal distribution with μ=7 and σ=3. This normal distribution assures that the starting lengths of the lightning rods are in the range of the generally expected values for HV substations (it is assumed that they are mounted on the appropriate pedestals). Initial population is then composed of a large number of these individuals (i.e. chromosomes).

Genetic algorithm is searching for the optimal solution to the external LPS design by minimizing a following fitness function:

$$Min\ f = w_1 \sum_{i\in\Gamma_c} \Xi_i + w_2 \sum_{j\in\Gamma_s} \Theta_j + w_3 \sum_{k\in\Gamma_r} \Psi_k + w_4 \cdot \Omega \tag{3}$$

that features four different criteria, as follows:

1. penalizing low stochastic efficiency of the LPS design, for each of the lightning classes, as follows:

$$\Xi_i = P(I_{min}^i) \cdot \begin{cases} A \cdot \mathrm{e}^{-\zeta(E_i - 0.5)} + B, & \text{if } E_i < 0.8 \\ A \cdot \mathrm{e}^{-\xi(E_i - 0.8)}, & \text{if } E_i \geq 0.8 \end{cases} \tag{4}$$

where E_i is the stochastic efficiency of the LPS design for the i-th class of lightning current amplitudes; Γ_c is a set of classes of lightning-current amplitudes, with class weights $P(I^i_{min})$ applied in such a manner as to emphasize those with low amplitudes; this particular weighting scheme entices GA to increase stochastic efficiency of those first few classes from which emanate the majority of shielding failures; additional following fixed parameters are utilized: A = 10000, B = 7000, ζ = 4, and ξ = 15,

2. penalizing short lightning rods in the vicinity of the protected AIS elements with many direct lightning strikes from the first class of lightning amplitudes, as follows:

$$\Theta_j = A \cdot \mathrm{e}^{(\ell_j^* - d_{jmin})/\kappa} \tag{5}$$

with

$$d_{jmin} = \min_{k\in\Gamma_r} \{d_{jk}\} \tag{6}$$

where d_{jk} is the shortest distance in 3D space between the protected element with the highest recorded number of direct strikes (within the j-th sector) and a k-th lightning rod; l_j* is the length of the rod that has been identified as being the closest to the protected element with the largest number of strikes (within the j-th sector); in addition, κ = 5; namely, the area occupied by the AIS equipment is subdivided into the Γ_s sectors (depending on the terrain topology, number of voltage levels and the actual station layout), and those lightning rods (in each sector) that are closest to the protected elements with the highest recorded number of direct lightning strikes are penalized in order to stimulate increasing their length (height); in other words, penalty is awarded to those lightning rods that are situated in the areas with the highest probability of shielding failures, as an incentive to increase their height,

3. penalizing both, excessively short, or long individual lightning rods from the set of Γ_r rods that form the variable part of the LPS, as follows:

$$\Psi_k = A \cdot \mathrm{e}^{\ \ \ell_k} + A \cdot \mathrm{e}^{\ell_k \ \ \gamma} \tag{7}$$

where parameter γ is used to provide a "soft" limit to the maximal length of the lightning rod, meaning that the penalty will exponentially increase beyond this limit; it is selected based on the voltage level of the substation and designer preferences; Fig. 1 graphically depicts this penalty function, for two different values of the limiting parameter,

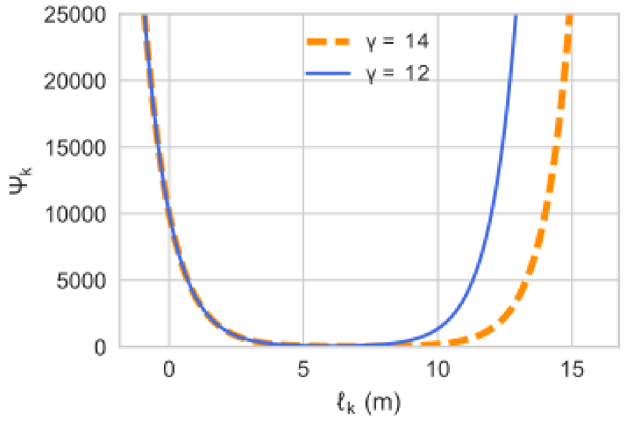

Fig. 1. Penalizing excessively short or long individuals.

4. penalizing excessive cumulative length of all lightning rods which form the variable part of the external LPS design, as follows:

$$\Omega = A \cdot \mathrm{e}^{-\delta/(N_r - 1)} \tag{8}$$

with

$$\delta = N_r \left[\mu + \sigma \cdot \Phi^{\ \ 1}(q)\right] - \sum_{i=1}^{Nr} \ell_i \tag{9}$$

where Φ^{-1} is the inverse cumulative distribution function of the standard normal distribution; a threshold of q = 0.8 is proposed, meaning that the penalty is increased as the cumulative length increases above the 80-th percentile of the associated distribution; imposing this penalty will prevent all rods to increase in length to the limit set by (7) and will balance out incentives offered by (5).

It can be seen that the fitness function employs a stochastic efficiency assessment of each individual within the population (i.e. each LPS design proposal) and assigns a value to each of them (the lower the fitness value is, the better is the stochastic efficiency of that particular LPS design). The stochastic LPS efficiency is obtained from applying the Monte-Carlo method. The four criteria mentioned above are mutually inclusive, balance each other (Σw=1) and are specifically designed for this particular problem. Together, they span a solution domain that is constructed from solid engineering principles of lightning protection, which enables the GA to effectively search for the optimal LPS design solution. A set of fixed parameters in the fitness function have been derived empirically, from a series of extensive numerical tests carried out on different substation dispositions and voltage levels.

Furthermore, three different operators are used in order to produce offspring [Goldberg, 1989]: crossover, mutation and reproduction. In the case of a crossover, two individuals (parents) are selected at random from the parental population and a two-point crossover principle is used in generating offspring (children). Only the first child is appended to the offspring population while the second child is discarded. A certain number of individuals (parents) undergo a mutation process, which assumes changing elements of its vector (with a mutation probability of 0.1) by a random value generated from the standard normal distribution.

Individuals undergoing reproduction are selected at random from the parental population, cloned and added to the offspring population. Selection of individuals from the parental population is carried out using a tournament selection principle, with three individuals participating in each tournament. A certain number of individuals with the best fitness (i.e. so-called "elite" individuals) are also retained in each epoch.

At the end of each epoch, the best individual within the population is selected and put into the "hall of fame" where it will remain unless a better individual is found in the coming generations. Only a small predefined number of individuals are allowed in the hall of fame at all times. The genetic algorithm loops through the previously described steps (fitness function calculation, selection, crossover, mutation, elitism) until convergence criteria is met.

The convergence of the genetic algorithm is checked using following independent criteria:

1. attained prescribed maximum number of generations,
2. no significant improvement in fitness between several successive generations (convergence),
3. attained prescribed overall (total) stochastic efficiency of the LPS system.

The algorithm terminates if any of the above mentioned criteria are met. The output of the genetic algorithm is the best individual from the last generation, along with a hall of fame. There is usually little variance between different solutions in the hall of fame. Moreover, differences in individual rod lengths of several decimeters, from different solutions in the hall of fame, are not crucial for the overall shielding design.

IV. STATION SHIELDING DESIGN OPTIMIZATION

In order to demonstrate the proposed GA approach to the external LPS design, an open-air transformer station (69 kV distribution substation) is employed, with a simplified layout provided in Fig. 2 (top view) and Fig. 3 (side view). Disposition of the station equipment, in terms of geometry, can be deduced from the principal drawing dimensions (in meters). This example has been appropriated from the IEEE Std. 998-2012 and the interested reader is at this point advised to consult this reference for further information.

The LPS of the substation consist of the fixed part which is not further optimized, but serves its shielding purpose as-is. These are, in this particular case, the four shield wires of the two transmission lines, along with the four two meter high lightning rods installed at the tops of the portal towers of these incoming transmission lines. Variable part of the external LPS design consists of six lightning masts, numerated and positioned according to Fig. 1. Only this variable part of the LPS is optimized.

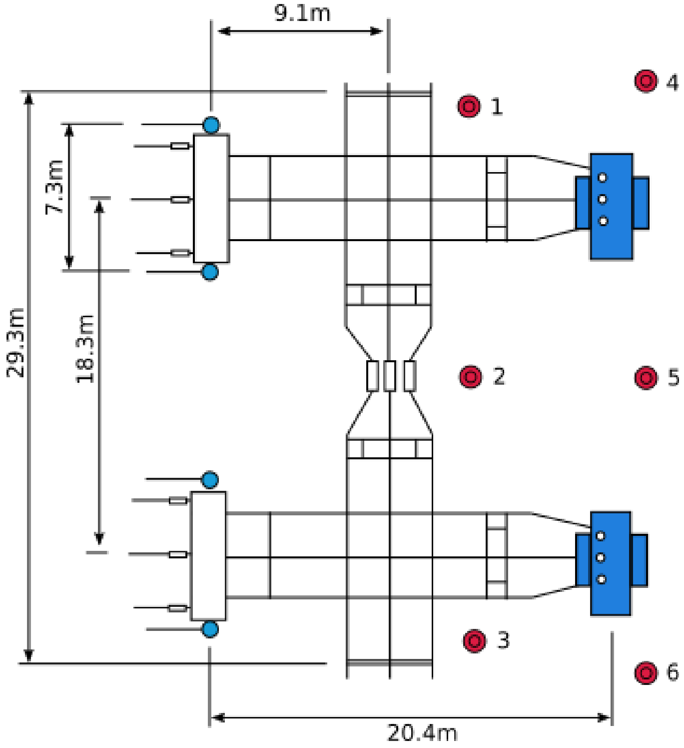

Fig. 2. Distribution substation design layout (top view).

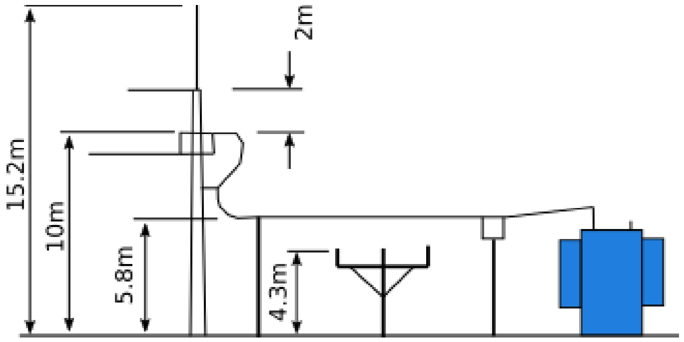

Fig. 3. Distribution substation design layout (side view).

In order to assess the effectiveness of the GA optimization approach, three different external LPS design solutions will be compared, as follows:

1. Variant A: fixed part of the external LPS plus variable part optimized by GA approach,
2. Variant B: fixed part of the external LPS plus variable part designed by the Eriksson's method of shielding,
3. Variant C: fixed part of the external LPS plus variable part consisting of all lightning masts of equal height of 20 meters.

Height of these lightning masts, six of them in total, will be optimized in order to provide maximum shielding with minimal investments. The stochastic efficiency of each LPS design will be compared for that purpose, along with the levels of design investments in terms of the total cumulative length of the applied lightning rods. Detailed scrutiny of the external LPS investment costs (in dollar terms) is not considered here due to the space concerns. Each of the three LPS designs is tested using a Monte-Carlo method with 200,000 individual lightning strikes, with amplitudes randomly drawn from the Log-Normal distribution in the range [1-80] kA; only 2.5% of amplitudes fall outside this range and these are associated with very large striking distances to be of practical importance. Furthermore, each LPS design will be compared in terms of the expected time window for a single shielding failure, which can be computed as follows:

$$\tau = \frac{1}{(1 - E_{tot}) \cdot N_g \cdot A_d} \tag{10}$$

where Ng stands for the ground flash density of the site in strikes per km^2 per year, while A$_d$ is the "collection area" of the AIS in km^2 (which is much larger that the actual substation area). Time windows in excess of 400 years are considered adequate for the lightning protection of substations at the high-voltage levels [Hofbauer, 1988]. This particular time window, however, should not be confused with the failure rates and partial shielding discussed in IEEE Std. 998-2012, nor with the mean time between failures of IEC 60071-2, although it can be seen as complementary to them.

Equal weights are applied on the four fitness function criteria. A population of 300 individuals is first instantiated and then maintained at that level throughout the entire evolution process. Tournament selection is used, with elitism, as already described. Around 300 children are produced in each generation, some of which come from the crossover (with probability 0.5), some from the mutation (with probability 0.3), and the rest from the reproduction (clones). Individual's fitness in each epoch is derived from 20,000 simulations. Hall of fame can hold only five best individuals at all times. A maximum of 30 epochs is allowed for the evolution process.

Table I present external LPS design characteristics obtained for the three different variants considered heretofore. Variant A is obtained using the proposed GA approach. Variant B is obtained by applying the Eriksson's method of shielding; see IEEE Std. 998-2012 for more information. Variant C is not optimized in any way and is given here only for reference purposes. It can be seen that the GA method and Eriksson's approach provide very similar results in terms of the heights of the masts needed for shielding coverage, although GA produces a design which uses the lowest cumulative length (meaning that it is most cost-effective). It is interesting to see how the GA has evolved a symmetric design of the lightning protection system.

TABLE I. SUBSTATION EXTERNAL LPS DESIGN CHARACTERISTICS

No.	**Variant A**	**Variant B**	**Variant C**
1	18 m	18 m	20 m
2	14 m	18 m	20 m
3	18 m	18 m	20 m
4	16 m	15 m	20 m
5	14 m	15 m	20 m
6	16 m	15 m	20 m
Σ	96 m	99 m	120 m

Table II presents computation results, regarding the class and overall (total) stochastic efficiency of the three different external LPS design proposals. The time window is computed for the ground flash density of 4 strikes per km^2 per year (approximately 40 thunderstorm days per year) and station collection area of 13870 m^2 for the Variants A and B, and 19380 m^2 for the Variant C. It can be seen here that the GA approach results with a design proposal which favors taller masts around the station periphery. It has slightly higher overall stochastic efficiency then Eriksson's solution, with shorter cumulative length of the employed shielding masts. This means that the GA proposed design is "better" than that which is obtained from the Eriksson's method. It should be mentioned that the LPS design evolved by the GA satisfies the criteria of the Eriksson's method of shielding. It can also be seen that the GA proposed design results with a higher stochastic efficiency for the first class of lightning amplitudes, which is of particular importance. The Variant C, which features the tallest overall and cumulative lightning masts, is the worst solution in terms of the time window for a single strike and design investment for the stochastic efficiency obtained. Namely, by increasing the height of the external LPS one increases the collection area of the station as well, which, consequently, has a negative impact on the lightning protection efficiency.

TABLE II. STOCHASTIC EFFICIENCY OF LPS DESIGNS

i-th class	**$I_{min} - I_{max}$**	**No. strikes**	**Stochastic efficiency (E_i)**		
			Var A	**Var B**	**Var C**
1	1.0 – 9.8	1682	0.9862	0.9841	0.9852
2	9.8 – 18.6	26915	0.9846	0.9845	0.9887
3	18.6 – 27.3	50371	0.9861	0.9858	0.9866
4	27.3 – 36.1	45275	0.9889	0.9878	0.9868
5	36.1 – 44.9	30926	0.9889	0.9899	0.9891
6	44.9 – 53.7	18867	0.9909	0.9909	0.9899
7	53.7 – 62.4	10984	0.9907	0.9907	0.9926
8	62.4 – 71.2	6290	0.9912	0.9926	0.9918
9	71.2 – 80.0	3598	0.9941	0.9917	0.9904
Overall Stochastic Efficiency (E_{tot})			0.9868	0.9866	0.9876
Time window for single strike (yrs)			1365	1345	1040

Additional comparison between here obtained LPS design characteristics and those obtained from different traditional and modern methods, presented in IEEE Std. 998-2012, is given in Table III. Although some methods have the same number of masts, their heights can vary significantly; see IEEE Std. 998-2012 for more information. The first two methods, which according to IEEE Std. 998-2012 permit some failure rate, use only a single mast (although a very tall one), while leader progression method (LPM) needs eight masts in total. All other methods, except for the EGM – Eriksson, agree on the number of masts being six, although heights of these masts are not the same with different methods. According to the way a stochastic efficiency has been here defined, none of the three variants considered had a hundred percent shielding coverage, notwithstanding that Eriksson's method had been considered to have a full shielding coverage (above the critical current level). In other words, with the stochastic efficiency as defined here, none of the methods would produce a hundred percent shielding efficiency, due to the fact that currents below the threshold level are generated in the Monte Carlo analysis (i.e. full statistical distribution of lightning currents is used).

TABLE III. COMPARISON OF DIFFERENT LPS DESIGNS

Method	**No. of masts required**
Fixed angle	1
Empirical	1
EGM - Mousa	6
EGM - RSM	6
EGM - Eriksson	7
CVM/FIFM	6
LPM (estimated)	8
LIT	6
Genetic Algorithm	6

It should be mentioned that the station's perimeter fence is not taken into account during the station shielding design, which is consistent with the treatment in IEEE Std. 998-2012. However, perimeter fence will have influence on the shielding design, due to the fact that it will provide some shielding for the apparatus and equipment at the station periphery. The genetic algorithm can automatically adapt to this fact and will evolve a design which accounts for the fence's position and height.

Generally speaking, the conservatism exercised by the protection engineer will influence the LPS design, giving rise to different designs produced by different people using the same method on the same substation; see IEEE Std. 998-2012 for more information. Another important aspect which needs to be accounted for is the keraunic level of the site which should influence the final shielding design. The GA produced design proposal can be of assistance, and can provide initial guidance to the engineer, during the planning phase of the station shielding design that needs to be based on one of the methods prescribed by the international standards.

V. CONCLUSION

This paper proposed an application of the genetic algorithm in designing optimal air-termination system of the HV substation external lightning protection system. It features a fully stochastic approach to the LPS efficiency assessment and an original GA fitness function developed from solid engineering principles of lightning protection design. Proposed GA approach offers a unique and valuable assistance to the LPS designer in optimally arranging elements of the air-termination system (of the external LPS) for obtaining maximum lightning shielding effects with minimum total investments.

It should be stated that the presented stochastic efficiency of the external LPS is still based on the EGM approach, which can be seen as a drawback to some extent, due to the fact that more advanced, physics based, non-conventional lightning attachment models exist today. Although these are not part of the present international standards, and research into their effectiveness for substation shielding is still ongoing, they can account for various aspects of the complicated lightning attachment process. Research into their implementation using GA is being actively pursued.

A somewhat rigid nature of the traditional approaches, implemented in present international standards, leaves little room for the cost-effective LPS optimization. This has been observed by several researchers and a statistical approach, which offers more freedom for subsequent optimization, has already been advocated on several occasions (starting from Sargent way back in 1972). Also, lightning data gathered by the lightning detection networks provide ample evidence for abolishing altogether the thunderstorm day as a measure of lightning activity (and with it the traditional keraunic maps). Design engineers still too often use simple rule of thumb approach and basic graphical-analytical analysis for the substation LPS design. Optimization has often been considered only as an afterthought and relegated to the ineffective trial-and-error approach.

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