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Propagation effects caused by stratified ground on electromagnetic fields of return strokes

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Abstract: In this paper the effect of stratified ground on the signature of ground-propagated electromagnetic fields radiated by lightning return strokes has been elucidated. Results are presented for two cases, stratified ground with two uniform layers and ground with continuously changing conductivity with depth. In addition to showing the anomalous propagation effects associated with stratified ground such as enhancement of certain frequencies when the conductivity of the upper layer is less than the bottom layer, the results demonstrate the possibility of remote sensing the seasonal variation of ground electrical conductivity using either narrow band or broad band measurement of electromagnetic fields.

1. Introduction

As electromagnetic fields propagate over finitely conducting ground, different frequencies are attenuated by different amounts. This leads to the modification of the signature of lightning generated radiation field, known in the scientific literature as propagation effects, as it propagates over finitely conducting ground (Cooray, 2003).

Knowledge on propagation effects is important both in characterising the interaction of lightning electromagnetic fields with structures and in the remote sensing of lightning current parameters from the electromagnetic radiation fields. Conversely, if the electromagnetic characteristics of a source are known, the propagation induced variation of the amplitude and the phase of the electromagnetic fields of the source as a function of frequency and distance can be utilized to probe the electrical characteristics of soil along the path of propagation.

In many studies dealing with the effects of finitely conducting ground on the signature of lightning generated electromagnetic fields the ground is treated as a homogenous semi-infinite half-space with a finite conductivity (Gardner, 1981;

LeVine et al., 1986; Cooray et al., 2000). In reality the conductivity of the soil may vary as a function of depth depending on the depth of different types of soil layers, soil temperature, and moisture content. Moreover, depending on the moisture content in soil the conductivity may vary more or less continuously with depth. Thus, the best way to approximate the real situation is to treat the ground as stratified.

In this paper we will study the propagation of electromagnetic fields over stratified ground. The analysis given in this paper will be confined to propagation distances of less than about 300 km. Therefore, the curvature of the earth and the ionospheric effects has been neglected in the analyses.

2. Theory

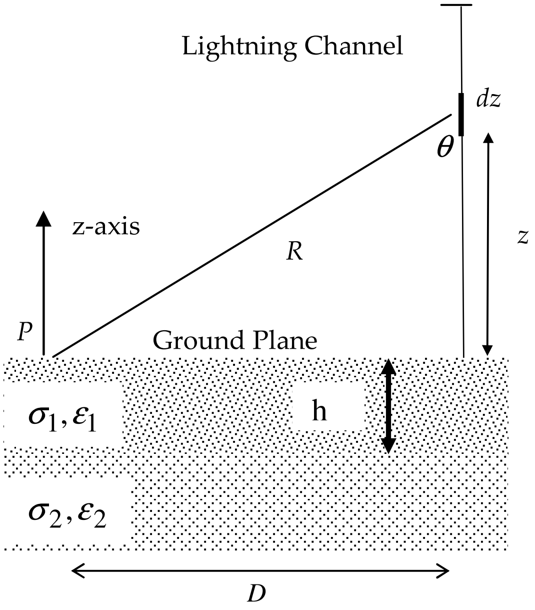


Figure 1: Geometry under consideration

The geometry relevant to the question under consideration is shown in Figure 1. Let us represent the ground as a stratum of thickness *h*, conductivity σ_1 and relative dielectric constant ε_1 below which the medium is semi-infinite with conductivity σ_2 and dielectric constant ε_2. The lightning channel is located at a distance D from the point of observation. In the calculation the lightning channel is assumed to be straight and vertical. The temporal variation of the current in the lightning channel at height z is represented by $I(z,t)$. First consider the case of perfectly conducting ground. The radiation field generated by the discharge, $E_z(t,D)$, at the point of observation is given by (Uman and MacLain, 1969)

$$E_z(t,D)=\frac{1}{2\pi\varepsilon_o}\int_0^H \frac{\sin^2\theta}{c^2R}\frac{dI(z,t-R/c)}{dt}dz \tag{1}$$

In this equation c is the speed of light in free space, ε_o is the electric permittivity of free space and H is the height of the return stroke. Equation 1 can be written in frequency domain as

$$e_z(j\omega,D)=\frac{1}{2\pi\varepsilon_o}\int_0^H \frac{\sin^2\theta}{c^2R}\ i(j\omega,z)j\omega e^{-j\omega R/c}dz \tag{2}$$

The frequency domain quantities $i(j\omega,z)$ and $e_z(j\omega,D)$ are related to the time domain quantities $I(z,t)$ and $E_z(t,D)$ through the Fourier transform

$$i(j\omega,z)=\int_o^\infty I(t,z)e^{-j\omega t}dt \tag{3}$$

$$e_z(j\omega,D)=\int_o^\infty E_z(t,D)e^{-j\omega t}dt \tag{4}$$

Over the stratified ground with finite conductivity (σ), the vertical electric field at ground level at distance D is given in frequency domain by the equation

$$e_{z,\sigma}(j\omega,D,\sigma)=\frac{1}{2\pi\varepsilon_o}\int_0^H \frac{\sin^2\theta}{c^2R}S(z,D,\sigma,j\omega)\ i(j\omega,z)j\omega e^{-j\omega R/c}dz \tag{5}$$

where $S(z,D,\sigma,j\omega)$ is the attenuation function that describes the effect of the stratified Earth. The effect of the stratified ground on the electromagnetic fields can be evaluated by numerically solving Equation (5) by substituting a suitable expression for the attenuation function given. However, Cooray (1987) simplified this expression by using the following arguments. Since one is interested in propagation effects, the section of the waveform which is of interest is that occurring within the first few microseconds. This is the case because the rapidly varying part of the waveform occurs within the first few microseconds. If the speed of propagation of the discharge front is about 10^8 m/s, the length of the channel that contributes to the radiation field during this time would not be larger than a few hundred metres. Thus, in Equation (5), the attenuation function $S(z, D, \sigma, j\omega)$ can be replaced by $S(0, D, \sigma, j\omega)$, the attenuation function corresponding to a dipole located at the lower end of the channel. With this approximation, Equation (5) can be transformed into the time domain to find an expression for $E_{z,\sigma}(t, D, \sigma)$ which is the vertical electric field over stratified ground. The result is

$$E_{z,\sigma}(t, D, \sigma) = \int_0^t E_z(t-\tau, D)\, S(0, D, \sigma, \tau)\, d\tau \qquad (6)$$

where $S(0, D, \sigma, t)$ is the inverse Fourier transformation of $S(0, D, \sigma, j\omega)$ and $E_z(t, D)$ is the radiation field over perfectly conducting ground. One can obtain $S(0, D, \sigma, t)$ through a direct Fourier transformation of $S(0, D, \sigma, j\omega)$. Wait (1962) derived the following expression for $S(0, D, \sigma, j\omega)$;

$$S(0, D, \sigma, jw) = 1 - j(\pi w)^{1/2} e^{-w}\, erfc(jw^{1/2}) \qquad (7)$$

$$w = -\frac{jk_o D}{2} \Delta_{eff}^2 \qquad (8)$$

In this equation Δ_{eff} is the effective normalized surface impedance of the stratified ground; it is defined as

$$\Delta_{eff} = \Delta_1 Q \qquad (9)$$

where

$$\Delta_1 = \frac{k_o}{k_1}\left[1 - \frac{k_o^2}{k_1^2}\right]^{\frac{1}{2}} \qquad (10)$$

$$Q = \frac{k_1 + k_2 \tanh(k_1 h)}{k_2 + k_1 \tanh(k_1 h)} \qquad (11)$$

with

$$k_1^2 = k_o\left[\varepsilon_{r1} - j60\sigma_1\lambda_o\right]^{\frac{1}{2}} \qquad (12)$$

$$k_2^2 = k_o\left[\varepsilon_{r2} - j60\sigma_2\lambda_o\right]^{\frac{1}{2}} \qquad (13)$$

$$k_o = \frac{2\pi}{\lambda_o} = \omega\left(\mu_o\varepsilon_o\right)^{\frac{1}{2}} \qquad (14)$$

In these equations, *erfc* stands for the complementary error function, μ_o is the magnetic permeability of free space, ε_r is the relative dielectric constant, ε_o is the electric permittivity of free space, λ_o is the free space wavelength, σ is the conductivity of the soil and $j = \sqrt{-1}$. The function $S(0, D, \sigma, t)$ can be obtained through a direct Fourier transformation of the function $S(0, D, \sigma, j\omega)$.

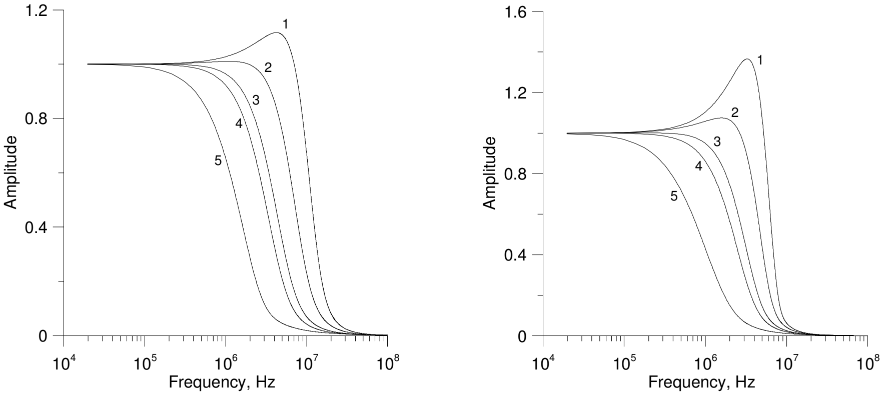
The equations given above are valid for a stratified ground with two layers. These equations can be modified to take into account stratified grounds with several layers or continuously stratified ground using the procedure outlined by Wait (1981).

In order to solve equations 6 an expression for $I(z,t)$, the spatial and temporal variation of the return stroke current, is required. Since no experimental data are available yet to quantify the way in which the return stroke current signature varies as a function of height, it is necessary to rely on return stroke models to obtain an expression for $I(z,t)$. In the literature one can find a wide variety of return stroke models and one can select any of these models for the study at hand. Here we utilize one of the simplest models, the modified transmission line model (Nucci et al., 1988). The current waveform at the channel base is represented by the analytical expression developed for the subsequent strokes by Nucci et al., (1990).

2. Results and discussion

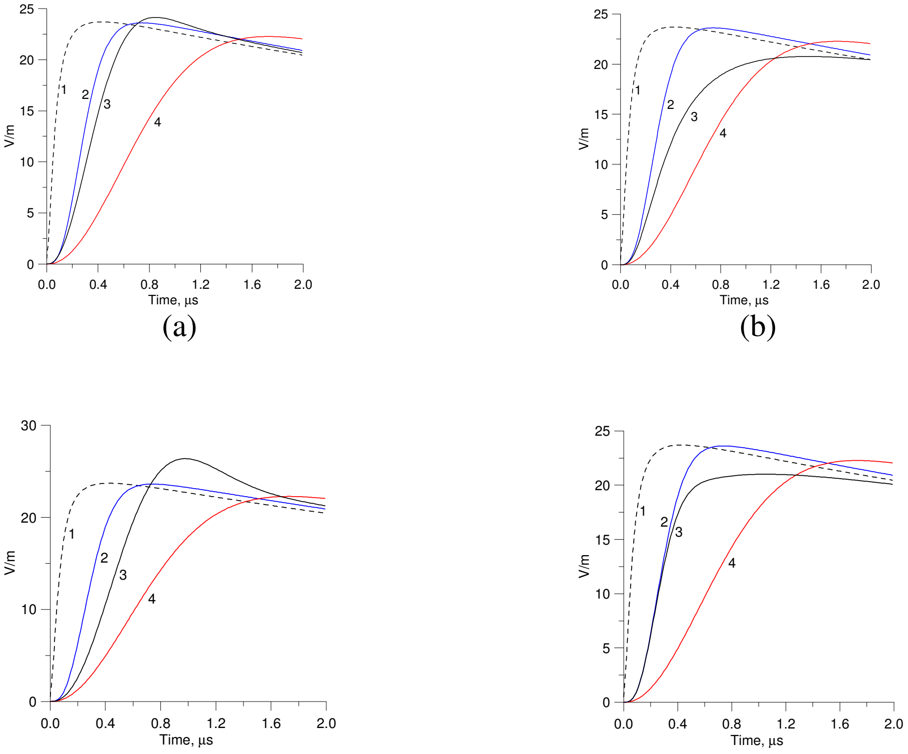
2.1 Stratified ground with two layers

The attenuation function in frequency domain for propagation distances of 10 and 100 km and for several values of σ_2 (while keeping σ_1 and h constant) is shown in Figure 2. Note that for $\sigma_1 < \sigma_2$ the attenuation function is larger than unity for certain frequencies. The reason for this is that when $\sigma_1 < \sigma_2$ the energy is being guided between the surface and the higher-conductivity boundary (Wait, 1962). Of course this enhancement of the field depends on the thickness of the upper layer. If the thickness of the upper layer is much large than the skin depth corresponding to a certain frequency then for that particular frequency the stratified earth behaves as homogeneous ground with conductivity σ_1. In the case of $\sigma_1 > \sigma_2$ the amplitude of the attenuation function is always less than unity and it decreases monotonically with increasing frequency.



(a) (b)

Figure 2: Attenuation function at (a) 10km and (b) 100 km as a function of frequency for stratified ground with two layers. The thickness of the upper layer is 2 m. 1) σ_1 =0.001 S/m, σ_2 =0.1 S/m. 2) σ_1 =0.001 S/m, σ_2 =0.01 S/m. 3) σ_1 =0.001 S/m, σ_2 =0.002 S/m. 4) σ_1 =0.001 S/m, σ_2 =0.001 S/m. 5) σ_1 =0.001 S/m, σ_2 =0.0001 S/m.



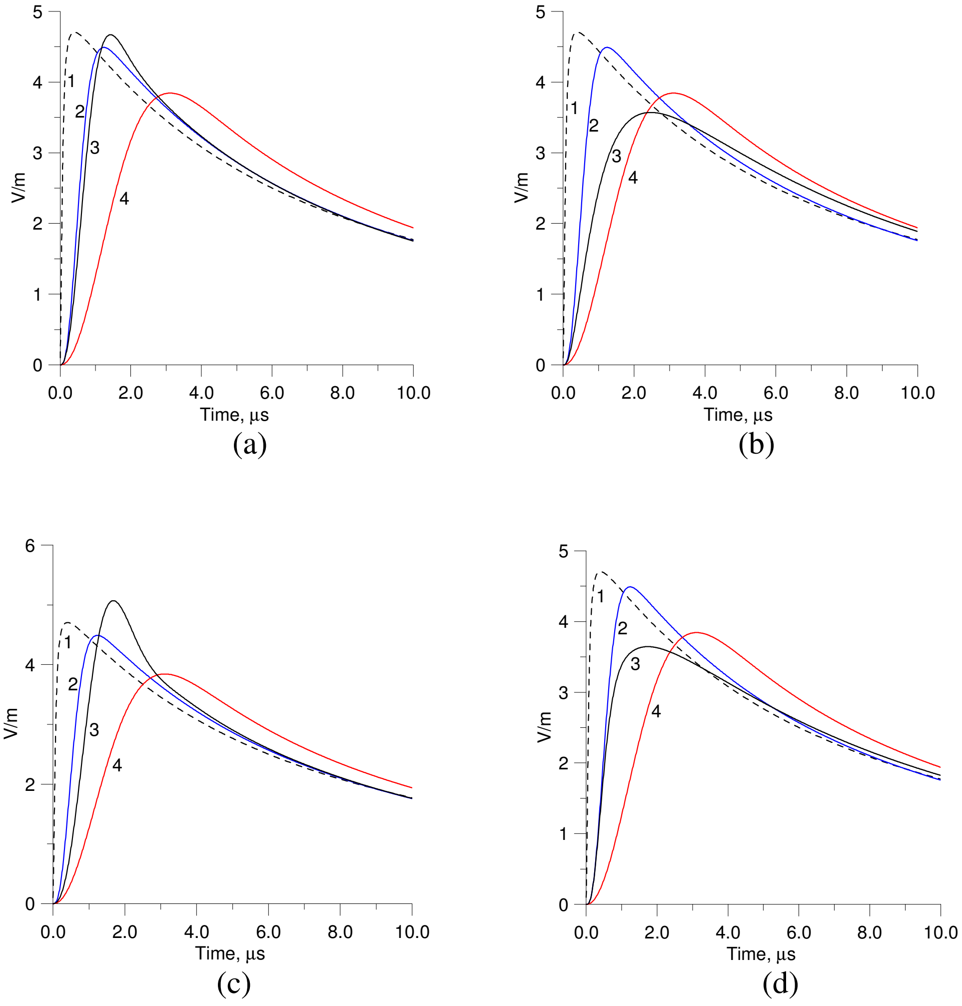
(a) (b)

(c) (d)

Figure 3: Electric field at 10 km generated by a lightning flash over stratified ground. In each figure, curve 1 is the field over perfectly conducting ground, curve 2 is the field over finitely conducting homogeneous

ground of conductivity 0.01 S/m and curve 4 is the field over finitely conducting homogeneous ground of conductivity 0.001 S/m. Curve 3 corresponds to the electric field over stratified ground. (a) $\sigma_1 = 0.001$ *S/m,* $\sigma_2 = 0.01$ *S/m, h = 2 m; (b) (b)* $\sigma_1 = 0.01$ *S/m,* $\sigma_2 = 0.001$ *S/m, h = 2 m; (c)* $\sigma_1 = 0.001$ *S/m,* $\sigma_2 = 0.01$ *S/m, h = 5 m; (d)* $\sigma_1 = 0.01$ *S/m,* $\sigma_2 = 0.001$ *S/m, h = 5 m.*

Figure 3 shows the electric field at 10 km generated by a lightning flash over stratified ground. For comparison purposes the electric fields that would be present over perfectly conducting ground and over homogeneous ground with conductivity equal to that of the upper or lower layer are also shown in the diagram. Figure 4 depicts the corresponding waveforms at 50 km distances. Note that when $\sigma_1 > \sigma_2$ the peak electric field is always lower than the one that is present over perfectly conducting ground. On the other hand, when $\sigma_1 < \sigma_2$ the electric field may contain a peak larger than its counter part over perfectly conducting ground.

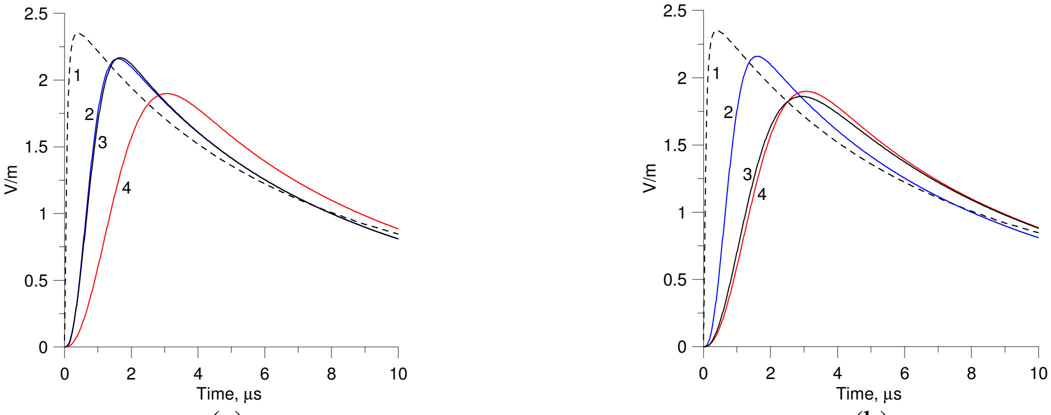


(a) (b)

(c) (d)

Figure 4: Electric field at 50 km generated by a lightning flash over stratified ground. In each figure, curve 1 is the field over perfectly conducting ground, curve 2 is the field over finitely conducting homogeneous ground of conductivity 0.01 S/m and curve 4 is the field over finitely conducting homogeneous ground of conductivity 0.001 S/m. Curve 3 corresponds to the electric field over stratified ground. (a) σ_1 *=0.001 S/m,* σ_2 *=0.01 S/m, h = 2 m; (b) (b)* σ_1 *=0.01 S/m,* σ_2 *=0.001 S/m, h = 2 m; (c)* σ_1 *=0.001 S/m,* σ_2 *=0.01 S/m, h = 5 m; (d)* σ_1 *=0.01 S/m,* σ_2 *=0.001 S/m, h = 5 m.*

Figure 5 depicts the electric field at 100 km over stratified ground with the conductivity boundary at a depth of 0.5 m. Note that for this shallow boundary, the behaviour for the stratified condition is very similar to the behaviour for the homogeneous condition with conductivity matching the value below the conductivity boundary.



(a)

(b)

Figure 5: Electric field at 100 km generated by a lightning flash over stratified ground. In each figure, curve 1 is the field over perfectly conducting ground, curve 2 is the field over finitely conducting homogeneous ground of conductivity 0.01 S/m and curve 4 is the field over finitely conducting homogeneous ground of conductivity 0.002 S/m. Curve 3 corresponds to the electric field over stratified ground. (a) σ_1 *=0.002 S/m,* σ_2 *=0.01 S/m, h = 0.5 m; (b)* σ_1 *=0.01 S/m,* σ_2 *=0.002 S/m, h = 0.5 m.*

2.2 Continuously stratified ground

As mentioned in the introduction the conductivity of the soil depends on the moisture content. More specifically, it has been shown that the moisture-related fractional change in soil electrical conductivity is proportional to (roughy) the square of the fractional change in water volume (Scheftic et al., this conference). The moisture content in soil can change in complicated manner that depends on short- and long-term variations in local rainfall. This makes the conductivity profile of soil as a function of depth more complicated. During rain the conductivity of upper soil increases and as the moisture penetrate into the ground the soil layers located at different depths will gradually become more conducting. So immediately after the rain the conductivity is high at the surface and it decreases with increasing depth. However, at the end of the rainy season the surface soil layers gradually dry up while the layer with high conductivity moves down. Thus the conductivity profile varies with time and this provides a possibility to utilize remotely-sensed electromagnetic fields at various frequencies to infer soil electrical parameters.

Figure 6 shows two conductivity profiles that are roughly in line with this description. In one profile (a) the conductivity is high at the surface while in the other (b) it increases with depth initially and then decreases again.

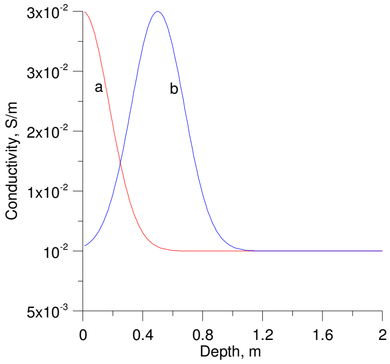


Figure 6: Two conductivity profiles selected in the calculations. In profile (a) the conductivity is maximum at the surface and in profile (b) it has moved to depth 0.5 m. The variation of conductivity with depth is assumed to be Gaussian.

Figure 7 shows how the amplitude of the electromagnetic field at two different frequencies varies as a function of distance when the ground is stratified according to Figure 6. Note that the shifting of the high-conductivity layer downwards produces a significant change in the amplitude of the electromagnetic field at both of the frequencies considered. The greatest fractional variation of amplitude between the two conductivity profiles occurs at about 1 MHz over the propagation range of 1-80 km.

Now let us consider the effect of these conductivity profiles on the time domain electromagnetic fields radiated by lightning return strokes. Figure 8 and 9 depict the electric field and the electric field derivative at 10 km, 20 km and 50 km for the two conductivity profiles shown in Figure 6. Note that there is a significant difference in the field derivatives in the two cases. However, the rise time differences are rather modest. These results indicate that one can use either single frequencies over the range of 10^5 to 10^7 Hz, the rise time of the radiation field or the time derivative of the radiation field to remote sense the seasonal variations of electrical conductivity of soil.

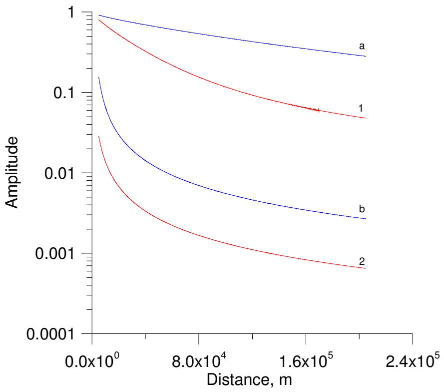
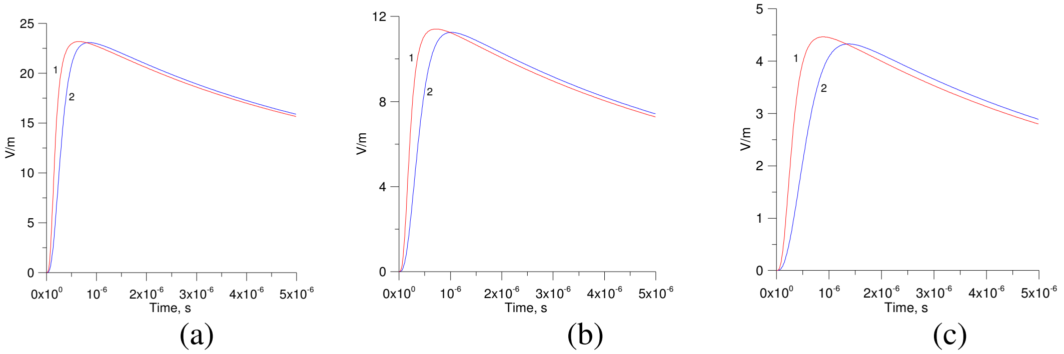
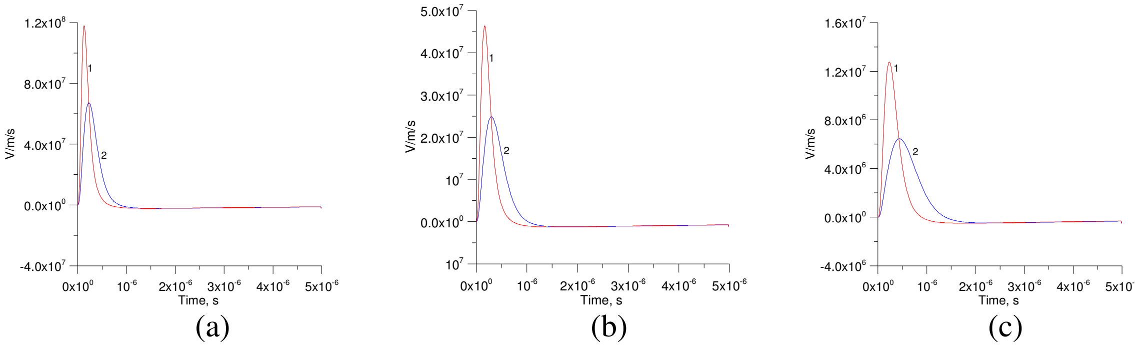


Figure 7: The amplitude of the electromagnetic field at different distances corresponding to conductivity profile (a) in Figure 5(curves marked 1 and 2) and conductivity profile (b) in Figure 5 (curves marked a, b). The frequencies corresponding to the curves are: 10^6 Hz (1 and a), and 10^7 Hz (2 and b).



(a) (b) (c)

Figure 8: Electric field corresponding to two conductivity profiles shown in Figure 5. (a) 10 km, (b) 20 km and (c) 50 km.



(a) (b) (c)

Figure 9: Electric field time derivative corresponding to two conductivity profiles shown in Figure 5. (a) 10 km, (b) 20 km and (c) 50 km.

3. Conclusions

In this paper we have elucidated the effect of stratified ground on the signature of electromagnetic fields radiated by lightning return strokes. Results are presented for two cases, one with ground having two conducting layers and the other with continuously varying conductivity. We show an anomalous propagation effects associated with stratified ground in the form of an amplitude enhancement at certain frequencies when the conductivity of the upper layer is less than the bottom layer. The results clearly demonstrate the possibility of remote sensing of seasonal variation of ground electrical conductivity using either narrow band or broad band measurement of electromagnetic fields.

4. References

Cooray, V., M. Fernando, T. Sörensen, T. Götschl, Aa. Pedersen, Propagation of lightning generated transient electromagnetic fields over finitely conducting ground, J. Atmos. Terres. Phy., 62, 583 – 600, 2000.
Gardner, R. L., Effects of propagation on lightning induced transient fields, Radio Sci., 16, 337-384, 1981.
Cooray, V., and S. Lundquist: Effects of propagation on the risetime and the initial peaks of radiation fields from return strokes, Radio Sci., vol. 18, 409-415, 1983.
Cooray, V., Effects of propagation on the return stroke radiation fields, Radio Sci., vol. 22, 757-768, 1987
Le Vine, D. M., L. Gesell, and M. Kao, "Radiation from lightning return strokes over a finitely conducting earth", J. Geophys. Res., Vol. 91, pp 11,897- 11,908, 1986.
Nucci, C. A., Mazzetti, C., Rachidi, F., Ianoz, M., On lightning return stroke models for LEMP calculations, paper presented at 19th International Conference on Lightning Protection, Graz, Austria, 1988.
Nucci, C. A., G. Diendorfer, M. A. Uman, F. Rachidi, M. Ianoz, C. Mazetti, Lightning return stroke models with specified channel base current: A review and comparison, J. Geophys. Res., 95, 20395-20408, 1990.
Uman, M. A. and D. K. McLain, Magnetic field of lightning return stroke, J. Geophys. Res., 74, 6899-6910, 1969.
Wait, J. R., Electromagnetic waves in stratified media, Pergamon Press, Oxford, England, 1962.
Wait, J. R., Wave propagation theory, Pergamon Press, Oxford, England, 1981.