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ELECTROMAGNETIC FIELDS INDUCED BY LIGHTNING STRIKING A PERFECT AND IMPERFECT CONDUCTIVE SURFACE

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Abstract— A derivation of lightning induced electromagnetic fields which originate from perfect and imperfect conductive surfaces upon being struck will be presented. The model presented here purposely downplays the physics of how image theory is employed to account for a charge which is in the presence of an imperfect conductive surface. In turn, it adopts an approach which focuses on the geometry that exists between the lightning channel and surface ground. In doing so, the proposed model formulates a solution that has minimized the complexity of the original problem while providing an approximation founded upon a geometric relationship. Hence, it has assumed that as a surface becomes less conductive the image channel which lies 180 degrees with respect to the return stroke begins to degrade. As a result, the subsequent image current which travels along this channel will decrease as well. A derivation of this degraded current will be presented along with how it influences the electric and magnetic fields. It will be shown that as the image current's magnitude is reduced the subsequent electromagnetic fields become smaller.

I. INTRODUCTION

Considerable research has been spent treating the surface in contact with the lightning channel as a perfect conductor. This ideology, in turn leveraged image theory to derive the resulting electromagnetic fields. However, recently more emphasis has been placed upon taking into account surfaces which are no longer considered perfect conductors.

In the literature, methods which are typically used to account for the lossy nature of conductive grounds have adopted the wavetilt formula, the Cooray formula, and the Cooray-Rubinstein formula. The wavetilt formula relates the Fourier transform of the horizontal electric field to that of the vertical electric field with the following expression [8],

$$W(j\omega) = \frac{E_H(j\omega)}{E_v(j\omega)} = \frac{1}{\sqrt{\varepsilon_r + \sigma / j\omega\varepsilon_0}}. \quad (1)$$

Where $E_H(j\omega)$ and $E_v(j\omega)$ are the Fourier transforms of the horizontal and vertical electric fields respectively with the relative permittivity of the soil ε_r, soil conductivity σ, the imaginary constant j, the angular frequency ω, and permittivity of the air ε_0. Although, this formula was found to be appropriate for remote observation points, it was deemed inapplicable for relatively close ranges [4].

As pointed out by Rubinstein [5], Weyl [6] expressed the results of the Sommerfeld integrals for the fields from a dipole over an imperfectly conductive surface as a group of plane waves that are reflected and refracted by the ground surface at incident angles with both real and imaginary constituents. Rubinstein [5] goes on to mention that if the surface ground is a relatively good conductor, these plane waves will refract at an angle that is approximately perpendicular to the surface of incident.

We know from image theory that a charge over an infinitely conductive ground has a perfect mirror image. This “mirror image” can be quantified by taking the charge’s spatial coordinates which are perpendicular to the surface and rotating or projecting them by 180 degrees. Taking the cosine of this angle gives rise to an image charge that is equal in magnitude but opposite in polarity. One can expand this idea to include the effect the surface conductivity has on the vertical and horizontal electric fields.

Rubinstein [5] introduced a new formula, presently known as the Cooray-Rubinstein formula that calculates the horizontal electric field above an imperfect conductor. This formula is broken into two terms, both of which assume a perfect conductive ground as shown by,

$$E_r(z=h,r) = E_{rp}(z=h,r) - H_{\Phi p}(z=0,r) \cdot \frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0 + \sigma / j\omega}}. \tag{2}$$

Where the first term is the horizontal electric field at a specified height h, the second term is the horizontal magnetic field at ground level multiplied by the surface impedance and p denotes a perfect conductor. Rubinstein [5] goes onto show that for large values of r, (2) reduces to the wavetilt formula. However, it was later shown by Shoory et al. [4] that the Rubinstein-Cooray formula for calculating the horizontal electric field is a valid approximation for close ranges but becomes inadequate for far ranges and poorly conducting grounds.

This paper is organized as follows: Section II presents Maxwell’s equations and provides the expressions that were used by [3] to derive the electric and magnetic field’s induced by a lightning strike. Image theory is introduced in Section III while the idea of the degraded image is introduced in Section IV. Section V introduces the electromagnetic fields from [3] to include the degraded current and Section VI provides the graphical comparison between the two. Finally, some conclusions are drawn in Section VII.

II. ELECTROMAGNETIC THEORY

In differential form Maxwell’s equations for a homogeneous, time variant, and linear medium can be written [1, 2] where $\mathbf{D}$ is the electric displacement, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{H}$ is the magnetic field strength, $\mathbf{J}$ is the current density, ρ is the charge distribution per unit volume, μ_0 the magnetic permeability and ε_0 is the electric permittivity.

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \tag{3}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{5}$$

$$\nabla \cdot \mu_0 \mathbf{H} = 0 \tag{6}$$

The preferred method, the dipole technique, leverages Maxwell’s equations to form a system of seven differential equations and seven unknowns given a known current distribution. From (3) through (5) the seven equations can be written as follows:

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0} \tag{7}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu_0 \frac{\partial H_x}{\partial t} \tag{8}$$

$$\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu_0 \frac{\partial H_y}{\partial t} \tag{9}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0 \frac{\partial H_z}{\partial t} \tag{10}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x + \varepsilon_0 \frac{\partial E_x}{\partial t} \tag{11}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y + \varepsilon_0 \frac{\partial E_y}{\partial t} \tag{12}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z + \varepsilon_0 \frac{\partial E_z}{\partial t} . \tag{13}$$

With the seven unknowns given by

$$E_x, E_y, E_z, H_x, H_y, H_z \ \& \ \rho .$$

Given equations (4) and (6), one can solve for the electric and magnetic fields in terms of the vector potential **A**. After the usage of some substitutions and vector identities one would obtain,

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} \tag{14}$$

and

$$\mu_0 \mathbf{H} = \nabla \times \mathbf{A} . \tag{15}$$

Given Lorentz Condition,

$$\nabla \cdot \mathbf{A} + \varepsilon_0 \mu_0 \frac{\partial \Phi}{\partial t} = 0 \tag{16}$$

one can solve for the potential Φ to obtain,

$$\Phi(r,t) = -\frac{1}{\varepsilon_0 \mu_0} \int_{0^-}^{t} (\nabla \cdot \mathbf{A}) d\tau + \Phi(t = 0^-). \tag{17}$$

Since at $t = 0^-$ there is no charge, the scalar potential must be zero as well. Therefore one can write the scalar potential in terms of the vector potential alone,

$$\Phi(r,t) = -c^2 \int_{0^-}^{t} (\nabla \cdot \mathbf{A}) d\tau \tag{18}$$

where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, with c equaling the speed of light. Finally, substitute the potential (18) into (14) to yield,

$$\mathbf{E} = c^2 \int_{0^-}^{t} \nabla(\nabla \cdot \mathbf{A}) d\tau - \frac{\partial \mathbf{A}}{\partial t} . \tag{19}$$

Equations (15) and (19) will be used along vector potential **A,**

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{v'} \frac{\mathbf{J}(\mathbf{r}_s, t - R/c) dv'}{R} \tag{20}$$

with the given current distribution,

$$i(z', t) = I_0 u(t - |z'|/v) \tag{21}$$

where u is the Heaviside function, to develop the expressions used to describe the electromagnetic fields induced by a lighting strike.

III. IMAGE THEORY

Image theory, in its current form, assumes that an image charge is in the presence of perfect conductor. By assuming the material is a “perfect conductor” allows one to account for all of the charge constituents. We can illustrate a method of images by considering the problem given in Figure 1 for a point charge q located at **y** relative to the origin, around which is centered a grounded conducting sphere of radius a.

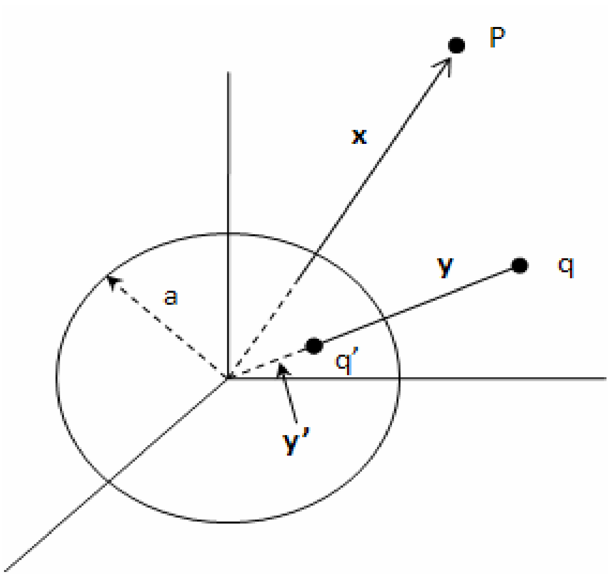

Figure 1. Conducting Sphere of Radius a, with Charge q, and Image Charge q'

One could think about image theory this way. Let's suppose you have a coherent light source and you shine it upon a dingy piece of metal. Some of the light will reflect back towards you but much of it will be lost due to refraction and/or absorption. Now take the same coherent light source and shine it towards a highly reflective mirror. You'll notice that most of the light, about 99%, will reflect back while only ~1% is lost due to refraction and/or absorption. Using the highly reflective mirror allowed you to account for the majority of light. That is, you're cognizant of where the light went because it reflected back towards you. In essence, by assuming we have a perfectly conducting material that acts like a mirror, allows one to project a particles image 180 degrees from the radial position where the original particle lies with respect to the conductive surface. This idea is illustrated in Figure 2.

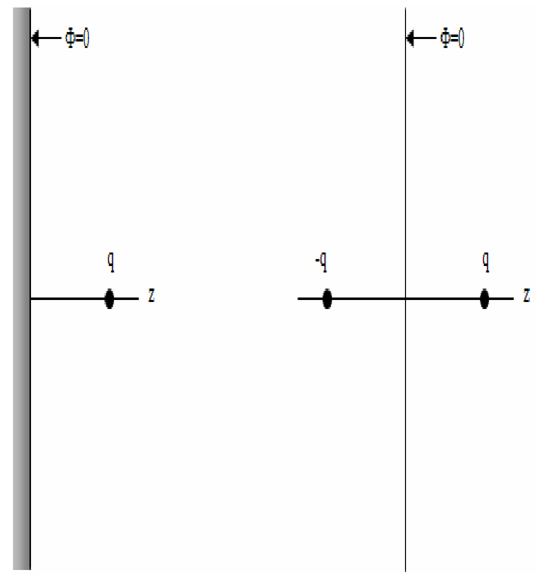

Figure 2. Solution by Method of Images

Figure 2 assumes that we're taking a very small slice of the spherical surface which houses the image charge as shown in Figure 1. If this section is small enough it can be modeled by a flat surface. With this idea in mind, we can expand this thought by applying this same basic principle to a lightning strike return stroke as shown in Figure 3.

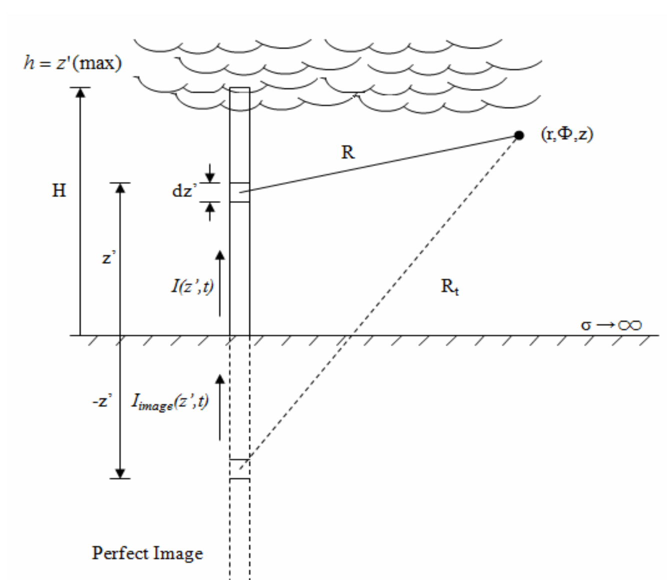

Figure 3. Application of Image Theory Used for the Lightning Channel

Figure 3 is a very high level illustration that depicts the lightning channel in contact with the earth's surface. The lightning channel can be represented by numerous adjoining steps, each of which may vary in length between 30 and 50 meters. Once the step leader meets up with a positively charged streamer emanating from the ground, a path for the return stroke has been created. As the return stroke travels upward along the lightning channel at velocity v [7], it imparts positive charges along the way which generates a current wavefront which propagates along the channel. Although this description brushes a broad stroke to the phenomenon that actually occurs, it can be further modified to encompass the idea of surfaces that are not perfect conductors as illustrated in Figure 4.

IV. THE DEGRADED IMAGE

In order to augment the approach used from image theory, this paper will introduce the idea of the degraded image. This ideology accommodates both perfect and imperfect projected images. With traditional image theory, it assumes that an image charge in the presence of a perfect conductor acts like a mirror. However, as the surface in contact with the lightning return stroke becomes less conductive, one can no longer assume that the image remains unchanged. In fact, one must concede to the idea that the entire image can no longer be projected in the same fashion as the image of a charge in the presence of a perfect conductor. With the adoption of this idea in place, it's logical to presume that as the grounding surface in contact with the lightning channel becomes less conductive, the image channel will become degraded as shown in Figure 4. If we assume that the lightning channel's width is finite, then any degradation to the channel will only occur along the *z*–axis. Subsequently, the magnitude of the current which travels along the image channel will decrease as well. Given the image channel has become degraded, some of the charges that were once part of this channel have become displaced. Since the conservation of charge must be preserved, these dislocated charges will now collect along the surface and induce currents which travel away from the channel. As a consequence, the image current now contributes less to the vertical electric and azimuthal magnetic fields while creating a horizontal electric field.

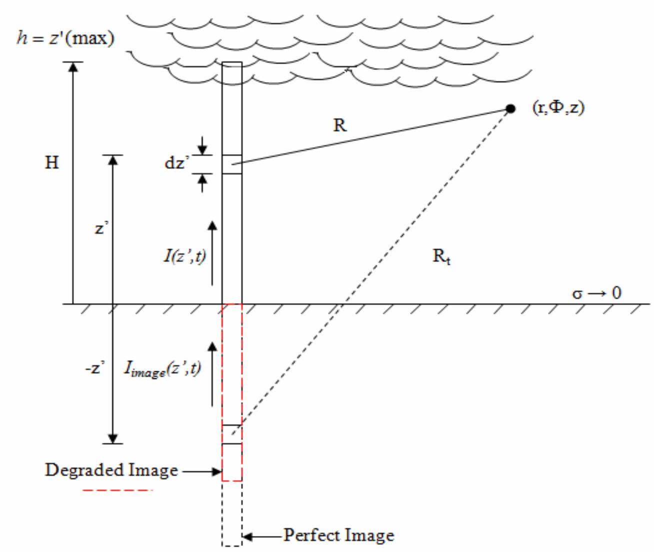

Figure 4. Application of the Degraded Image Used for the Lightning Channel

Figure 4 illustrates the how the degraded image will change with respect to the perfect image as the conductivity, σ of the surface decreases. As the surface becomes a perfect insulator, the magnitude of the degraded image channel approaches zero. This methodology has purposely downplayed the physics of how image theory is employed to account for a charge which is in the presence of an imperfect conductive surface. In doing so, this model formulates a solution that has minimized the complexity of the original problem while providing an approximation founded upon a geometric relationship. Knowing how the image channel is affected by the surface conductivity allows one to develop an equivalency between the two by exploiting the geometry of Figure 4. Since the contribution from the magnitude of the perfect image current is generally known, this value can be scaled to account for a changing conductivity as shown in Figure 5.

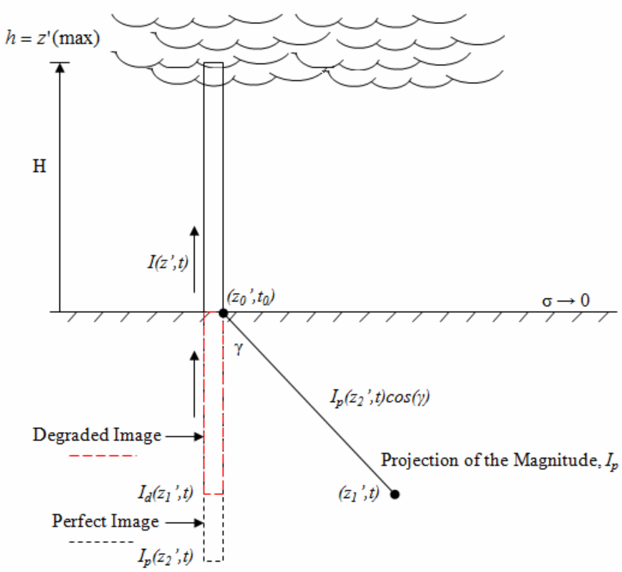

Figure 5. Projection of the Degraded Current Used for the Lightning Channel

As Figure 5 illustrates, the contribution from the degraded image can be quantified by taking the magnitude of the current which travels along the perfect image and scaling it by a factor which accounts for the loss. This can be realized by taking the projection of the perfect image's current and rotating it along the r–z axis until it shares the same z component as the degraded image. Do so will allow one to associate the perfect image with the degraded image by multiplying the scaling factor, $\cos(\gamma)$. Therefore, we can write the degraded current in terms of the perfect current such that,

$$I_d = I_0 \cos(\gamma) \tag{22}$$

where I_d is the degraded image current, I_0 is the perfect image current and the cosine of the angle, γ accounts for the loss. In order to maintain the conservation of charge, one must arrive at the notion that the image charge constituents that are no longer present within the degraded image channel must now be present elsewhere. With this in mind, we can infer that these image charges collect along the surface and induce currents which create electromagnetic fields in addition to those originally accounted for in [3]. Although these surface currents spread out radially, their contributions can be approximated by formulating two distinct currents, each of which lie on opposite sides of the channel-ground interface. In principle, these two surface currents represent the summation of each of their respective radial constituents, thus formulating a viable approximation to those present. We can describe these surface currents with the following,

$$I_s = \frac{I_0}{2}\left(1 - \cos(\gamma)\right) \tag{23}$$

where I_s equals the surface current. By summing (22) and (23) we can now describe the contributions made by the image current for both perfect and imperfect conductive surfaces. The total image current can now be written as,

$$I_I = I_0 \cdot \cos(\gamma) + \frac{I_0}{2}\left(1 - \cos(\gamma)\right) \tag{24}$$

where I_I equals the image current from the degraded and surface elements.

V. ELECTROMAGNETIC FIELDS

The electric and magnetic fields from [3] were found to be

$$E_z(r,0,t) = \frac{I_0}{2\pi\varepsilon_0}\left[\frac{-th+\frac{2h^2}{v}+\frac{r^2}{v}}{\left(h^2+r^2\right)^{3/2}} - \frac{1}{rv} - \frac{r^2}{c^2\left(h^2+r^2\right)^{3/2}\left(\frac{1}{v}+\frac{h}{c\sqrt{h^2+r^2}}\right)}\right] \tag{25}$$

and

$$H_\Phi(r,0,t) = \frac{I_0}{2\pi}\left[\frac{h}{r\sqrt{h^2+r^2}} + \frac{r}{\frac{c}{v}\left(h^2+r^2\right)+h\sqrt{h^2+r^2}}\right] \tag{26}$$

where,

$$h = \beta\frac{(ct-\beta z)\pm\sqrt{(z-\beta ct)^2+\left(1-\beta^2\right)r^2}}{1-\beta^2} \tag{27}$$

and the quantity $\beta = v/c$ is the ratio of the current propagation speed along the lightning channel and the speed of light.

However, we can now modify these equations to account for surfaces in contact with lightning strikes that are of all types of conductivity. By incorporating (22),(24) into (25) and (26) allows one to calculate the electromagnetic fields that would transpire in the presence of both perfect and imperfect conductive surfaces. These fields can now be described by (28) and (29) such that

$$E_z(r,0,t) = \frac{I_0}{4\pi\varepsilon_0}\left[\frac{-th+\frac{2h^2}{v}+\frac{r^2}{v}}{\left(h^2+r^2\right)^{3/2}} - \frac{1}{rv} - \frac{r^2}{c^2\left(h^2+r^2\right)^{3/2}\left(\frac{1}{v}+\frac{h}{c\sqrt{h^2+r^2}}\right)}\right] + \frac{I_d}{4\pi\varepsilon_0}\left[\frac{-th+\frac{2h^2}{v}+\frac{r^2}{v}}{\left(h^2+r^2\right)^{3/2}} - \frac{1}{rv} - \frac{r^2}{c^2\left(h^2+r^2\right)^{3/2}\left(\frac{1}{v}+\frac{h}{c\sqrt{h^2+r^2}}\right)}\right] \tag{28}$$

and

$$H_\Phi(r,0,t) = \frac{I_0}{4\pi}\left[\frac{h}{r\sqrt{h^2+r^2}} + \frac{r}{\frac{c}{v}\left(h^2+r^2\right)+h\sqrt{h^2+r^2}}\right] + \frac{I_1}{4\pi}\left[\frac{h}{r\sqrt{h^2+r^2}} + \frac{r}{\frac{c}{v}\left(h^2+r^2\right)+h\sqrt{h^2+r^2}}\right]. \tag{29}$$

VI. RESULTS

A. Illustration of the Magnetic Field in contact with a surface of varying conductivity

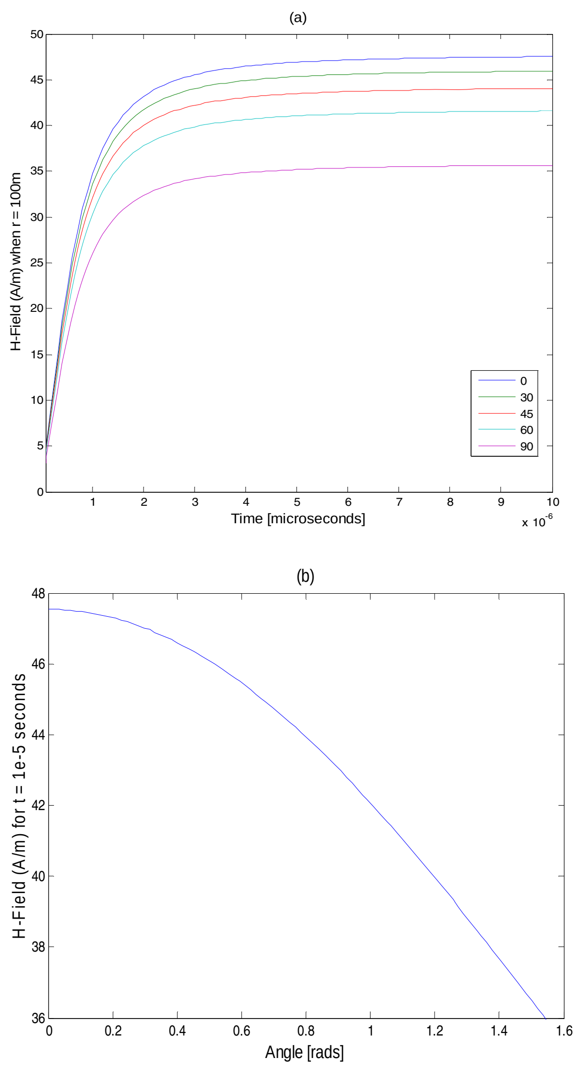

Figure 6. (a) Depicts the Magnetic Fields when r = 100m. (b) Depicts the Magnetic Field for angles 0 – pi/2 when t = 10^{-5} seconds and r = 100m

B. Illustration of the Vertical Electric Field in contact with a surface of varying conductivity

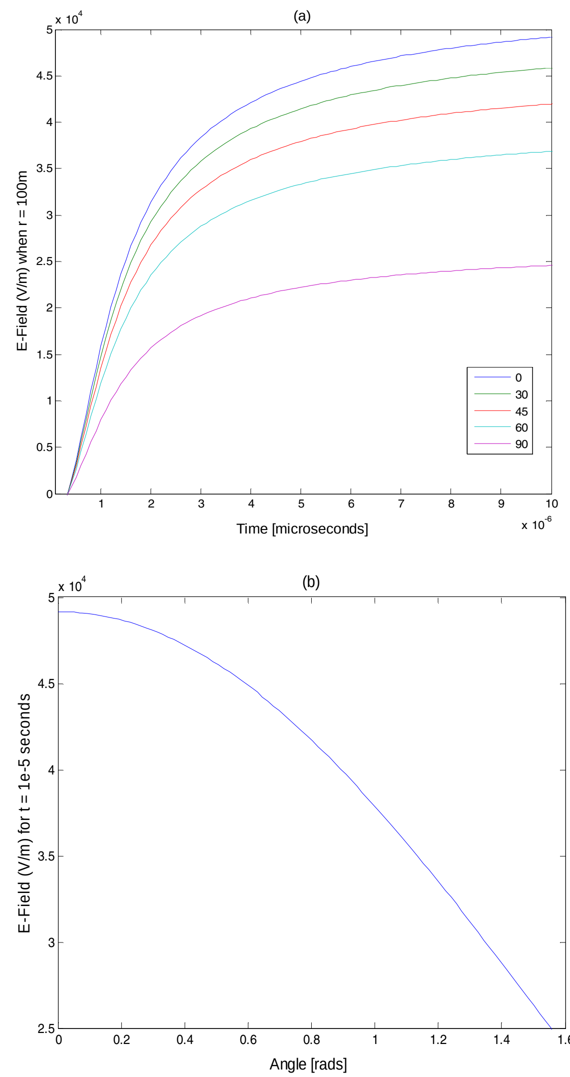

Figure 7. (a) Depict the Electric Fields when r = 100m. (b) Depicts the Electric Field for angles 0 – pi/2 when t = 10^{-5} seconds and r = 100m

As each graph illustrates, the effect to the magnetic and electric fields when γ = 0, 30, 45, 60, and 90 degrees is appreciable. One readily observes that as the angle gamma increases the subsequent fields decrease. However, it is important to note that the vertical electric field was slightly more affected by the surface conductivity than that of the horizontal magnetic field. This is primarily due to the increased presence of surface currents which add to the magnetic field but not to the vertical electric field.

VII. CONCLUSION

The magnetic and electric fields which originate from a lightning strike return stroke in contact with both perfect and imperfect conductive surfaces have been presented. The model presented here purposely downplayed the physics of how image theory is employed to account for a charge which is in the presence of an imperfect conductive surface. In turn, it adopted an approach which focused on the geometry that exists between the lightning channel and surface ground. In doing so, the proposed model formulated a solution that minimized the complexity of the original problem while providing an approximation founded upon a geometric relationship. As the results showed, the effects to the electromagnetic fields due to a decrease in ground conductivity were appreciable.

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